

Charge-exchange resonances and restoration of the Wigner SU(4)-symmetry in heavy and superheavy nuclei

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Abstract. Energies of the giant Gamow-Teller and analog resonances - E_G and E_A , are presented, calculated using the microscopic theory of finite Fermi system. The calculated differences $\Delta E_{G-A} = E_G - E_A$ go to zero in heavier nuclei indicating the restoration of Wigner SU(4)-symmetry. The calculated ΔE_{G-A} values are in good agreement with the experimental data. The average deviation is 0.30 MeV for the 33 considered nuclei for which experimental data is available. The ΔE_{G-A} values were calculated for heavy and superheavy nuclei up to the mass number $A = 290$. Using the experimental data for the analog resonances energies, the isotopic dependence of the difference of the Coulomb energies of neighboring nuclei isobars analyzed within the SU(4)-approach for more than 400 nuclei in the mass number range of $A = 3 - 244$. The Wigner SU(4)-symmetry restoration for heavy and superheavy nuclei is confirmed. It is shown that the restoration of SU(4)-symmetry does not contradict the possibility of the existence of the "island of stability" in the region of superheavy nuclei.

1 Introduction

The possible existence of spin-isospin resonance was first discussed in the works of Ikeda, Fujii, and Fujita in the middle of the 1960s as an attempt to explain the observed suppression effect of Gamow-Teller (GT) nuclei - transitions [1–3]. They called this state the Gamow-Teller resonance (GTR), in analogy with analog resonance (AR) or Fermi resonance.

The energies and probabilities of exciting these resonances were calculated later for a large group of spherical nuclei [4–6] within the theory of finite Fermi systems (TFFS) [7]. The first results were published in 1972 [4], several years before the experimental observation of the GTR. It was assumed at first that as a spin-flip state, the GTR should be located above the AR by the average energy of spin-orbit splitting E_{ls} , and that its width should be on the order of E_{ls} . It was found later from our calculations that the energy splitting between the GTR and AR was not equal to E_{ls} , but it decreased for heavy isotopes. The effect of decreasing the gap between the GTR and AR energies was first noted in 1973 as a result of calculations for more than 70 isotopes [5, 6]. We concluded that the Wigner SU(4)-supersymmetry [8] must be restored in heavy nuclei because GTR and AR belongs to the same supermultiplet in this approach, and their main parameters are the same. Since at the time of GTR prediction its energies have not been measured for different nuclei, then at that time it was impossible to solve the question of experimental verification of the hypothesis of Wigner SU(4)-symmetry restoration in heavy nuclei.

At present time there are three methods to test this hypothesis by comparing the calculated and experimental nuclear data. The first - from the analysis of the degeneration of the Gamow-Teller and analog (AP) resonances, as in this case, both of the resonance must belong to the same supermultiplet according to SU(4)-approach. The second possibility is related to the realization for nuclear masses the Franzini-Radicatti relation [9] following from the SU(4)-theory. The third one - associated with the analysis of the Coulomb energies of nuclei and their isotopic dependence [10]. Analysis of these three possibilities and restoration of Wigner supersymmetry was considered recently in [11].

In this paper compares the results of our calculations of the energy difference ΔE_{G-A} between GTR - E_G and AR - E_A with experimental data and investigated restoration of Wigner SU(4)-symmetry up to superheavy nuclei with $A = 290$. The isotopic dependence of the difference of the Coulomb energies of neighboring nuclei isobars also analyzed for more than 400 nuclei with known experimental data in the range of mass numbers $A = 3 - 244$. In connection with the restoration of Wigner supersymmetry in heavy nuclei, it becomes uncertain interpretation of the spin-orbit splitting and the associated shell structure and therefore, the possibility of the existence of the "island of stability" in the region of superheavy nuclei [12]. This problem is also discussed in this paper.

2 Method of calculation

The Gamow-Teller resonance and other charge-exchange excitations of nuclei are described in the TFFS with a system of equations for the effective field [7]. For the GT ef-

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fective nuclear field, we obtain a system of equations that, in the λ -representation has the form:

$$V_{\lambda\lambda'} = V_{\lambda\lambda'}^\omega + \sum_{\lambda_1\lambda_2} \Gamma_{\lambda\lambda'\lambda_1\lambda_2}^\omega A_{\lambda_1\lambda_2} V_{\lambda_2\lambda_1} + \sum_{\nu_1\nu_2} \Gamma_{\lambda\lambda'\nu_1\nu_2}^\omega A_{\nu_1\nu_2} V_{\nu_2\nu_1},$$

$$V_{\nu\nu'} = \sum_{\lambda_1\lambda_2} \Gamma_{\nu\nu'\lambda_1\lambda_2}^\omega A_{\lambda_1\lambda_2} V_{\lambda_2\lambda_1} + \sum_{\nu_1\nu_2} \Gamma_{\nu\nu'\nu_1\nu_2}^\omega A_{\nu_1\nu_2} V_{\nu_2\nu_1},$$

$$A_{\lambda\lambda'}^{(p\bar{m})} = \frac{n_\lambda^{\bar{p}}(1 - n_{\lambda'}^p)}{\varepsilon_\lambda^{\bar{p}} - \varepsilon_{\lambda'}^p + \omega}, \quad A_{\lambda\lambda'}^{(n\bar{p})} = \frac{n_\lambda^{\bar{n}}(1 - n_{\lambda'}^n)}{\varepsilon_\lambda^{\bar{n}} - \varepsilon_{\lambda'}^n - \omega},$$

$$V_{GT}^\omega = e_q \sigma \tau_+ \quad (1)$$

where n_λ and ε_λ are, respectively, the occupation numbers and energies of λ -states. The subscripts ν are used for the l -forbidden part of the interaction. The system of secular equations (1) for the charge-exchange excitations of nuclei is obtained from the more general [13], including the effective changes of pairing gap Δ in the ph - and pp -channels. In our case we use the condition $d_{pn}^{(1)} = d_{pn}^{(2)} = 0$ assuming that the effects of changes of the pairing gap in the external field are negligible, which is justified in this case for the external fields with zero diagonal elements of [7]. Parameters of single-particle states and their wave functions are calculated in the shell model for neutrons and protons separately. Pairing is taken into account in the single-particle structure by the replacement $\varepsilon_\lambda \rightarrow E_\lambda = \sqrt{\varepsilon_\lambda^2 + \Delta_\lambda^2}$ as in [14], where the energy Δ_λ is calculated separately for neutrons and protons. The quasi-particle effective charge e_q is $e_q = 1$ for Fermi ($\tau\tau$) transitions and $e_q < 1$ for Gamow-Teller ($\sigma\tau$) transitions. The energies of charge-exchange excitations are defined as the eigenvalues ω_i of secular equations (1), with the most the collective energy ($\omega_{GTR} > \omega_i$), and the maximum matrix element $M_{GTR}^2 \approx e_q^2 3(N - Z)$.

In the calculations we use a local nucleon-nucleon δ -interaction Γ^ω in the Landau-Migdal form with the coupling constants f_L' and g_L' of the isospin-isospin ($\tau\tau$) and the spin-isospin ($\sigma\tau$) quasi-particle interaction with $L = 0$. For the ($\tau\tau$) coupling constant the value $f_0' = 1.35$ was used, taken from comparison of calculating energy splitting between the analog and anti-analog isobaric states (IS) with the experimental data for the large number of nuclei [15]. For the ($\sigma\tau$) coupling constant g_0' in the previous calculations [16] we used value $g_0' = 1.22$ obtained by comparing the difference between the GTR (E_{G1}) and IS (E_{G2}) energies with the experimental data for nine Sb isotopes [17]. Considering that the absolute values of the constants can vary in different approaches, the obtained ratio $g_0'/f_0' = 0.90 \pm 0.03$ is model independent in the TFFS.

The energies of GTR and AR were calculated also in the self-consistent TFFS (used a simplified version of [18] with the local interaction and $m^* = m$), and in its approximate model version [19] in which solutions were obtained in analytical form for collective IS. For this purpose, we neglect the l -forbidden terms in (1) and assume the constant effective field when collective modes excited. For the energy differences $\Delta E_{G-A} = E_G - E_A$ the solution with

$V(r) = \text{const}$, normalized on the energy E_{ls} , has a form for the nuclei with $\Delta E > E_{ls}$ ($x = \Delta E/E_{ls} > 1$):

$$y_0 = \frac{\Delta E_{G-A}}{E_{ls}} \approx (g_0' - f_0')x + b \frac{1 + bg_0'}{g_0'x(1 + c_A/x^2)} \quad (2)$$

where $\Delta E = (4/3)\varepsilon_F(N - Z)/A$ ($\varepsilon_F \approx 40$ MeV).

$$E_{ls} = \sum_{\lambda_1\lambda_2} n_{\lambda_1}(1 - n_{\lambda_2})\varepsilon_{\lambda_1\lambda_2}^{ls} / \sum_{\lambda_1\lambda_2} n_{\lambda_1}(1 - n_{\lambda_2})$$

$$b = (2/3)(1 - (2A)^{-1/3}), \quad c_A = 0.8A^{-1/3}$$

The average energy of spin-orbit splitting E_{ls} is a parameter in Eq. (2) that can be calculated either from the single-particle scheme of GT spin-flip transitions, as was done in [6, 18], or can be obtained phenomenologically by comparing calculated ΔE_{G-A} energies and the experimental data [16]. The dependence of E_{ls} for heavy nuclei with the number of neutrons $N > 80$ we used the parameterization:

$$E_{ls} = 20N^{-1/3} + 1.25 \quad (\text{MeV}), \quad (3)$$

obtained as in [20] but substituting A for N . Shell effects in the region of lighter nuclei were considered. Such behavior according formula (3) corresponds to decreasing the E_{ls} value and to restoration of the Wigner SU(4)-symmetry in heavy nuclei.

Equation (2) is also applicable for heavy and super-heavy nuclei, because for them the value of $x = \Delta E/E_{ls}$ is larger and the accuracy of the calculations should be better.

3 Results and discussion

Energy differences between the Gamow-Teller and analog resonances $\Delta E_{G-A} = E_{GTR} - E_{AR}$ were calculated using Eq. (2) for 33 nuclei: ^{48}Ca , $^{60,64}\text{Ni}$, ^{71}Ga , ^{76}Ge , ^{82}Se , $^{90,91,92,94}\text{Zr}$, ^{93}Nb , $^{94,96,97,98,100}\text{Mo}$, ^{115}In , $^{112,114,116,117,118,119,120,122,124}\text{Sn}$, $^{128,130}\text{Te}$, ^{127}I , ^{136}Xe , ^{150}Nd , ^{169}Tm and ^{208}Pb (the initial target nuclei), for which experimental data are available (we used experimental data from [16, 19]). Calculated and experimental dependences of relative energy $y(x) = \Delta E_{G-A}/E_{ls}$ on dimensionless parameter $x = \Delta E/E_{ls}$ are presented in Fig. 1. The leftmost and rightmost points correspond to ^{60}Ni and ^{208}Pb with $x = 0.52$ and $x = 2.15$, respectively. The difference between the calculated and experimental results $\Delta\varepsilon = |\Delta E_{G-A}^{\text{calc}} - \Delta E_{G-A}^{\text{exp}}|$ is 0.38 MeV for ^{60}Ni and less than 0.10 MeV for ^{208}Pb , indicating that the accuracy of calculations improves for heavy nuclei. The r.m.s. deviation of the calculations according to formula (2) for 33 listed nuclei from the experimental data is $\delta(\Delta\varepsilon) \leq 0.30$ MeV which is comparable to the accuracy of the E_{GTR} experimental data [17].

Fig. 1 also presents calculations for the nuclei located at the "line of beta-stability" (LBS), which was determined by the formula

$$Z_\beta = A/(2 + 0.015A^{2/3}), \quad (4)$$

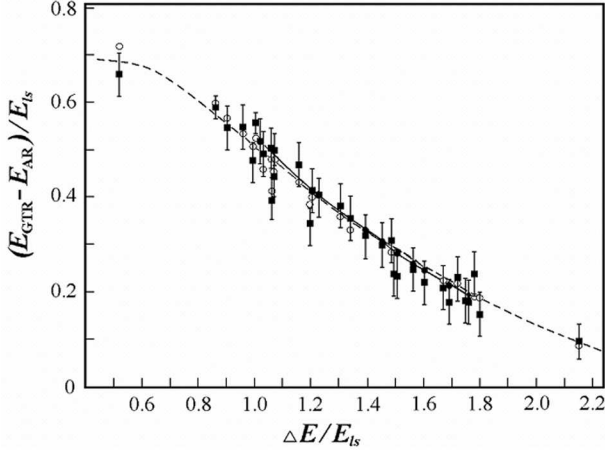


Figure 1. Dependence of the dimensionless energy difference between the GTR and AR $y(x) = (E_{\text{GTR}} - E_{\text{AR}})/E_{\text{Is}}$ on parameter $x = \Delta E/E_{\text{Is}}$ calculated by Eq. (2) (open circles) and experimental data (black squares). Black dots connected by the line represent the values calculated for Sn isotopes; the dashed line, those calculated with the E_{Is} values obtained with Eq. (3) for nuclei located on the beta-stability line determined by Eq. (4).

derived in [21] from the condition $\partial M/\partial Z = 0$ (with constant A), using well-known formula for the binding energy in the liquid-drop model of the nucleus. Here, Z_β corresponds to the minimum mass of nucleus for each isobaric chain.

4 Analog resonances and Coulomb displacement energies in SU(4)-approach

If the analog and Gamow-Teller resonances belong to the same supermultiplet, so the AR energies should be described in the framework of SU(4)-theory. The analysis of the applicability of the SU(4)-approach was performed in [10], where the possibility of the description of the difference ΔE_C between the Coulomb energies of neighboring isobar nuclei within the SU(4)-theory was analyzed. However, this analysis was performed in [10] only for nuclei with $A < 60$; for this reason, an unambiguous conclusion cannot be made. We analyzed the ΔE_C values for more than 400 nuclei for which experimental data are known in the mass number interval $A = 3-244$ (we used the data presented in [22]). As in [10, 22], we used the two parametric formula

$$\Delta E_C = E_C(A, Z+1) - E_C(A, Z) = a \frac{Z}{A^{1/3}} + b. \quad (5)$$

For all nuclei with $A = 3-244$, we obtained $a = 1416$ and $b = -698$ keV with the standard deviation $\delta E < 100$ keV. Deformation was taken into account phenomenologically as in [22], by introducing the correction to $\Delta E_C^{\text{def}} = \Delta E_C^{\text{sph}} - \delta E_C^{\text{def}}$ with the deformation parameters β_2 and β_4 from [23]. In the SU(4)-scheme, four types of ground states of nuclei belong to different supermultiplets: (i) (Z-even, N-even) nuclei belong to the $(T_Z, 0, 0)$ supermultiplet; (ii) (Z-even, N-odd) nuclei belong to the $(T_Z, 1/2,$

$1/2)$ supermultiplet; (iii) (Z-odd, N-even) nuclei belong to the $(T_Z, 1/2, -1/2)$ supermultiplet; and (iv) (Z-odd, N-odd) nuclei belong to the $(T_Z, 1, 0)$ supermultiplet where the isospin $T_Z = (N - Z)/2$ and the energy ΔE_C is considered as the difference between the energy of the ground state of the (A, Z) nucleus and the energy of excitation of the analog resonance in the $(A, Z+1)$ nucleus taking into account the energy of β -decay Q_β . Correspondingly, taking into account the mass difference $\Delta M = M_n - M_H = 782.35$ keV, we obtain the relation $b = \beta - \Delta M$ for the parameter b from Eq. (4), where the parameter β in the SU(4)-scheme should depend on the supermultiplet of the ground state. In particular, for nuclei with even Z , i.e., for cases (i) and (ii), the equality $\beta = 0$ [10] should be satisfied; this equality is really observed: the average deviation of the β value from zero is 80 keV. The most interesting cases are Z-odd nuclei, for which the SU(4)-scheme gives the dependence $\beta = \alpha/T_Z$ on the isospin T_Z , where the parameter α is different for N-even and N-odd nuclei [10]. The analysis shows the inverse dependence $\beta \approx 83/T_Z$ keV for nuclei with odd Z values according to the SU(4)-approach [10]. However, we were not able to obtain different α values for different supermultiplets because of insufficient data on ΔE_C for odd-odd nuclei.

Nevertheless, analyzing the experimental data on the energies ΔE_C for more than 400 nuclei, one can state that the observed functional dependence corresponds to the SU(4)-theory.

5 Energies of Gamow-Teller and analog resonances in heavy and superheavy nuclei

Equations (2) and (3) are also valid for heavy and superheavy (SH) nuclei and provide even better results because the parameter $x = \Delta E/E_{\text{Is}}$ is larger in this case and the conditions of the model approach applicability for solving TFFS equations (1) are better.

Figure 2 shows the results of the calculations of the absolute value $\Delta E_{\text{G-A}}$ as a function of the mass number for isotopes with $A > 140$ located on the line of beta stability. These isotopes with $Z_\beta(A)$ were found for each isobaric chain by the minimum mass of the nucleus from the experimental data [24]. The microscopic calculations of energy differences between the Gamow-Teller and analog resonances for the ^{257}Fm , ^{271}Sg , ^{280}Ds , and ^{290}Lv isotopes with allowance for the single-particle structure, as in [25], are also presented. According to Eq. (1), these calculations are approximate because the deformation of nuclei was taken into account phenomenologically, as in [22]. Meanwhile, the consistent inclusion of deformation should affect the single-particle spectrum. However, the effect of deformation on the energy of spin-orbit splitting, which determines the position of the Gamow-Teller resonance, is small. It was found that the energies $\Delta E_{\text{G-A}}$ calculated by Eq. (1) for four heavy nuclei differ within 0.1 MeV from those calculated by Eq. (2). As is seen in Fig. 2, the Gamow-Teller and analog resonances are also degenerate for heavy nuclei. However, the $\Delta E_{\text{G-A}}$ values microscopi-

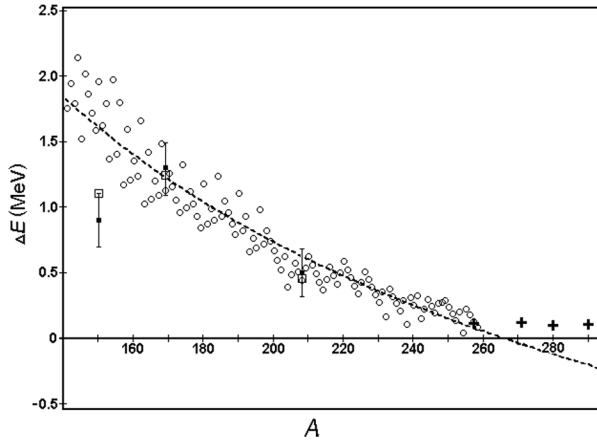


Figure 2. Energy difference ΔE_{G-A} versus mass number A (open squares) calculate by Eq. (2) and (closed squares) obtained experimentally for the ^{150}Nd , ^{169}Tm , and ^{208}P isotopes. Circles are the calculations for nuclei located on the line of beta stability from [24]. The dashed line is calculated for nuclei located on the line of beta stability determined by Eq. (3). Crosses are calculations for the ^{257}Fm , ^{271}Sg , ^{280}Ds , and ^{290}Lv nuclei according to Eq. (1).

cally calculated according to Eq. (1) are somewhat larger than those obtained by Eq. (2) for nuclei on the line of beta stability because of a more correct calculation of the energy E_{ls} from the single-particle level scheme. The analysis of spin-orbit splitting in superheavy nuclei performed in [26] within the generalized self-consistent method of the energy density functional demonstrated that the consistent variation of the parameters of the spin-orbit interaction slightly affects the energies of spin-orbit splitting; i.e., this quantity is stable.

Thus, taking into account degeneracy in the matrix elements of the Gamow-Teller and analog resonances [27], one can conclude that a decrease in the energy difference ΔE_{G-A} between the Gamow-Teller and analog resonances in heavy nuclei is due to the restoration of SU(4)-symmetry and both resonances belong to the same Wigner supermultiplet together with the ground state of the initial (A, Z) nucleus.

6 Conclusions

The calculated in this work values of the Gamow-Teller and analog resonance energy differences ΔE_{G-A} were found to be in good agreement with the experimental data. The root-mean-square deviation is 0.30 MeV for the 33 considered nuclei with known experimental data. The convergence of GTR and AR energies for the group of heavy nuclei with $Z \geq 100$ on the beta-stability line was investigated. The ΔE_{G-A} values were calculated for heavy and superheavy nuclei with mass numbers up to $A = 290$. On the base of observed degeneration of Gamow-Teller and analog resonances in heavy nuclei and predicted in superheavy, the Wigner SU(4)-restoration is confirmed. This allows to describe the heavy nuclei properties more con-

fidently using SU(4) theory, especially for mass relations [28].

The developing relationship for the masses of the nuclei, and the analysis of the Franzini-Radikatti [9] relation for nuclei masses, resulting from the SU(4)-theory, and which was performed for several times [29, 30], confirms that these relations works better in heavier nuclei. Also the analysis of the Coulomb displacement energies, using the SU(4)-approach allow to describe the mass and energies of superheavy nuclei with good accuracy.

So, it has been shown that Wigner supersymmetry is restored in heavy nuclei. As a result, the interpretation of the energy of spin-orbit splitting and the corresponding shell structure, as well as thereby the possibility of the existence of the "island of stability" in the region of superheavy nuclei, become indefinite. Our analysis of the degeneracy of the Gamow-Teller and analog resonances involves the ratio $x = \Delta E/E_{ls}$, which increases in heavy nuclei with the energy $\Delta E \sim (N - Z)/A$ even at the constant value E_{ls} . Here, E_{ls} is the average energy of spin-flip single-particle transitions within spin-orbit doublets (2), which decreases with an increase in the neutron excess. According to estimate (2), the energy E_{ls} tends to a finite value in heavy nuclei and does not vanish. The microscopic calculations for superheavy nuclei (see Fig. 2) confirm that E_{ls} is greater than zero and even increases slightly when approaching the "island of stability"

Thus we may conclude, that the restoration of Wigner supersymmetry in heavy nuclei does not contradict the possibility of the existence of the "island of stability" in the region of superheavy nuclei [31].

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